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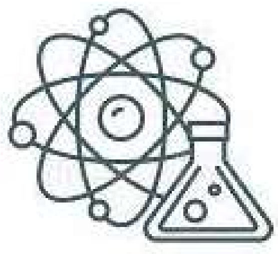

Mathematics

preparing students for exams

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1. A box contains 3 red balls and 2 green balls. Two balls are drawn one after the other without replacement. If a green ball is drawn first, find the probability that the second ball is red.

G = Green ball draw
R = Red ball draw

P(G) = 2/5
P(R) = 3/5

$P(R|G) = P(R \cap G)/P(G) = (2/5 \times 3/4)/(2/5) = 3/4$

Answer: 3/4

2. A fair die is rolled once. What is the probability of obtaining a number greater than 4?

a) □ 1/6 b) □ 1/3 c) □ 1/2 d) □ 2/3

We know that,

P(number i rolled in a fair die) = 1/6, i = 1, 2, 3, 4, 5, 6

P(rolled a number greater than 4) = P(rolled 5) + P(rolled 6) = 1/6 + 1/6 = 2/6 = 1/3

Answer: b)

3. Given that sin(θ) = 1/2 and 0° ≤ θ ≤ 180°, find the value of 2cos(θ).

a) ☐ $\sqrt{3}$ **b) ☐ $-\sqrt{3}/2$** **c) ☐ 1/2** **d)☐-1/2**

We know that,

sin^2**(θ) +** cos^2**(θ) =1**

So,

$(\frac{1}{2})^2$ **+** cos^2**(θ) =1 ⇔ ¼ +** cos^2**(θ) =1 ⇔** cos^2**(θ) =1 -1/4 ⇔** cos^2**(θ) =3/4 ⇔**

cos(θ) = $\pm\frac{\sqrt{3}}{2}$

2cos(θ) = $\pm\sqrt{3}$

Answer: a)

4. An arithmetic sequence has first term a_1 = 4 and common difference d = 3. Find the 15th term.

In an arithmetic sequence $\mathbf{a_n = a_1 + (n - 1) \times d}$.

If $a_1 = 4$ and $d = 3$ then $a_n = 4 + (n - 1) \times 3$.

$a_{15} = 4 + (15 - 1) \times 3 = 46$

Answer: 46

5. Find the equation of the line passing through the points A(2, −1) and B(6, 7)

1. Equation of a line

$$y = mx + b$$

2. Compute the slope with the points (x1, y1) and (x2, y2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7+1}{6-2} = 2$$

3. Compute the equation of the line

$$y = mx + b \Leftrightarrow 7 = 2x6 + b \Leftrightarrow b = -5$$
$$y = 2x - 5$$

Answer: y = 2x - 5

6. Which of the following functions is strictly increasing for all real x different from zero?

a)□ $f(x) = -x^2$ b) □ $f(x) = x^3$ c) □ $f(x) = -2x + 1$ d) □ $f(x) = 5 - x^2$

We compute de first derivative of all functions, and if the derivative is always positive for all x different from zero, the function is strictly increasing for all x different from zero.

$f(x) = -x^2$ $f'(x) = -2x$

$f(x) = x^3$ $f'(x) = 3x^2 > 0$

$f(x) = -2x + 1$ $f'(x) = -2 < 0$

$f(x) = 5 - x^2$ $f'(x) = -2x$

Answer: b)

7. Solve the inequality: $x^2 - 4x - 5 \leq 0$

We know that $x^2 - 4x - 5 = (x-5)(x+1)$

	$-\infty$	-1		5	$+\infty$
$(x-5)(x+1)$	+	0	-	0	+

$x \in [-1, 5]$

Answer: $x \in [-1, 5]$

8. A circle has radius 7 cm. What is its area? (Use π = 22/7)

a) ☐ 44 cm² **b) ☐ 154 cm²** **c) ☐ 308 cm²** **d) ☐ 616 cm²**

We Know that,

$$Acircle = \pi r^2$$

A = 22/7 x 49 = 22 x 7 = 154

Answer: $\boldsymbol{b})$

9. Given the function $f(x) = x^2 - 2x - 3$, find the coordinates of the vertex.

Vertex coordinates = (-b/2a; f(-b/2a))

$x_{vertex} = 2/2 = 1$

$y_{vertex} = 1^2 - 2 \times 1 - 3 = -4$

vertex coordinates : (1, - 4).

Answer: (1, - 4)

10. Let z = 3 +3i. Write z in polar form

1. $\rho = \sqrt{a^2 + b^2} = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$

2. $tg(\theta) = \frac{b}{a} = \frac{3}{3} = 1$ $\theta = \frac{\pi}{4}$ (because θ is an angle of 1st quadrant)

z= $\rho e^{(\theta i)} = 3\sqrt{2}e^{(\frac{\pi}{4}i)}$

Answer: $z = 3\sqrt{2}e^{(\frac{\pi}{4}i)}$

11. One of the solutions of the equation $x^2 - 4x + 13 = 0$ is:

a) ☐ 2+3i **b) ☐ 2+4i** **c) ☐ -2+3i** **d) ☐ -2-3i**

We use the resolving formula to solve the equation x^2 - $4x$ + 13 = 0

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Leftrightarrow x = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 13}}{2 \times 1} \Leftrightarrow x = \frac{4 \pm \sqrt{16 - 52}}{2} \Leftrightarrow$$

$$x = \frac{4 \pm \sqrt{-36}}{2} \Leftrightarrow x = \frac{4 \pm \sqrt{36i^2}}{2} \Leftrightarrow x = \frac{4 \pm 6i}{2}$$

$$\Leftrightarrow x = 2 \pm 3i$$

Answer: a)

12. The function f(x) = $\frac{e^{3x}-5x^2}{4x}$ is given. Write the expression of the first derivative of f(x).

f(x) = $\frac{e^{3x}-5x^2}{4x}$

f´(x) = $\frac{(e^{3x}-5x^2)'4x-(e^{3x}-5x^2)(4x)'}{(4x)^2} = \frac{(3e^{3x}-10x)4x-(e^{3x}-5x^2)4}{16x^2} =$

$\frac{(3e^{3x}-10x)x-(e^{3x}-5x^2)}{4x^2} = \frac{(3xe^{3x}-10x^2)-e^{3x}+5x^2}{4x^2} = \frac{(3xe^{3x}-5x^2)-e^{3x}}{4x^2} = = \frac{(3x-1)e^{3x}-5x^2}{4x^2}$

Answer: f'(x) = $\frac{(3x-1)e^{3x}-5x^2}{4x^2}$

13. The sum of the first 10 terms of the arithmetic sequence $u_n = 5n + 1$ is:

a) ☐ 260 **b) ☐ 275** **c) ☐ 285** **d) ☐ 270**

We have for an arithmetic sequence,

$$S_n = \frac{u_1+u_n}{2}n$$

1. Calculate u_1 and u_{10}

$u_1 = 5 \times 1 + 1 = 6$
$u_{10} = 5 \times 10 + 1 = 51$

2. Calculate S_{10}

$S_{10} = (6 + 51)/2 \times 10 = 57 \times 5 = 285$

Answer: c)

14. Let $f(x) = x^3 + \ln(2x)$, $x>0$. Compute $f''(1)$.

$f(x) = x^3 + \ln(2x)$

$f'(x) = 3x^2 + \frac{2}{2x} = 3x^2 + \frac{1}{x}$

$f''(x) = 6x - \frac{1}{x^2}$

$f''(1) = 6 \times 1 - \frac{1}{1} = 6 - 1 = 5$

Answer: $f''(1) = 5$

15. For the function h(θ) = 2 - 3cosθ, the value of θ when h = 2 is:

a) ☐ $\frac{\pi}{6}$ b) ☐ $\frac{\pi}{3}$ c) ☐ $\frac{\pi}{4}$ d) ☐ $\frac{\pi}{2}$

When h = 2, we have

$$2 = 2 - 3\cos\theta \Leftrightarrow 3\cos\theta = 0 \Leftrightarrow \cos\theta = 0 \Leftrightarrow \theta = \frac{\pi}{2}$$

Answer: d)

16. Find the vertical and horizontal asymptotes of the function g(x) = $\frac{2x-1}{x+3}$.

Let g(x) = $\frac{2x-}{x+3}$

(x + 3) = 0 ⇔ x = - 3

Vertical asymptote: x = − 3

(2x -1) : (x + 3) = 2 + $\frac{-7}{x+3}$

Horizontal asymptote: y = 2

Answer: Vertical asymptote: x = −3, Horizontal asymptote: y = 2

17. The first derivative of the function $f(x) = \ln(2x) + 5e^{4x}$ at $x = 1$ is:

a) ☐ $1+20e^4$ b) ☐ $2+20e^4$ c) ☐ $1/2+20e^4$ d) ☐ $1+5e^4$

$f(x) = \ln(2x) + 5e^{4x}$

$f'(x) = \frac{1}{x} + 20e^{4x}$

$f'(1) = \frac{1}{1} + 20e^{4} = 1 + 20e^{4}$

Answer: a)

18. Let $p(x) = 4x^2 + \ln(x)$, $x > 0$. Determine the x-coordinate of the inflection point, if it exists.

We have,

$p(x) = 4x^2 + \ln(x)$

$p'(x) = 8x + \frac{1}{x}$

$p''(x) = 8 - \frac{1}{x^2}$

$p''(x) = 0 \Leftrightarrow 8 - \frac{1}{x^2} = 0 \Leftrightarrow \frac{8x^2-1}{x^2} = 0 \Leftrightarrow 8x^2 - 1 = 0 \; and \; x^2 \neq 0 \Leftrightarrow x^2 = 1/8 \Leftrightarrow x = \pm \frac{\sqrt{2}}{4}$

Como $x > 0$, $x = \frac{\sqrt{2}}{4}$

Answer: $x = \frac{\sqrt{2}}{4}$

19. Find the exact value of θ (θ>0) such that 2 + 3sinθ = 5. The value of cosθ is:

a) ☐ $\frac{\sqrt{3}}{2}$ **b)** ☐ $\frac{1}{2}$ **c)** ☐ **0** **d)** ☐ $-\frac{1}{2}$

2 + 3sinθ = 5 ⇔ 3sinθ = 5 – 2 ⇔ sinθ = 3/3 = 1 ⇔ θ = $\pi/2$

$$\cos\left(\frac{\pi}{2}\right) = 0$$

Answer: c)

20. The complex number z = $5e^{(\frac{\pi}{4}i)}$ in algebraic form is:

a) ☐ **5+5i** **b)** ☐ $5\frac{\sqrt{2}}{2}+5\frac{\sqrt{2}}{2}i$ **c)** ☐ $\sqrt{5}+\sqrt{5}\,i$ **d)** ☐ $5\sqrt{2}+5\sqrt{2}i$

We have,

z = $5e^{(\frac{\pi}{4}i)}$

In algebraic form,

$$z = 5\cos\left(\frac{\pi}{4}\right) + 5sin\left(\frac{\pi}{4}\right)i = 5\,\frac{\sqrt{2}}{2} + 5\,\frac{\sqrt{2}}{2}\,i$$

Answer: $\boldsymbol{b})$