



Mathematics

preparing students for exams

1. A fair six-sided die is rolled twice. What is the probability that the sum is 7?

- ☐ **1/6**
- ☐ **1/12**
- ☐ **1/9**
- ☐ **1/36**

Each die has 6 sides, so rolling two dice gives:

$6 \times 6 = 36$ total

We know that all the results we can observe are:

| | | | | | |
|--------------|--------------|--------------|--------------|--------------|--------------|
| (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (5,6) |
| (6,1) | (6,2) | (6,3) | (6,4) | (6,5) | (6,6) |

If we count all the events where the sum is equal to 7, we have 6 events.

$P(\text{sum is equal to seven}) = 6/36 = 1/6$

Answer: $1/6$

2. If $P(A) = 0.7$, $P(B) = 0.4$, and $P(A \cap B) = 0.2$, what is $P(A \cup B)$?

- ☐ 0.9
- ☐ 0.8
- ☐ 0.7
- ☐ 1.1

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.7 + 0.4 - 0.2 = 0.9$$

Answer: 0.9

3. For the function $f(x) = 3x^2 - 12x + 5$, the coordinates of the vertex are:

- ☐ (2, -7)
- ☐ (2, 3)
- ☐ (-2, 7)
- ☐ (1, -4)

We know that,

vertex coordinates = $(-b/2a; f(-b/2a))$

$a = 3$ and $b = -12$

$$x_{\text{vertex}} = 12/6 = 2$$

$$y_{\text{vertex}} = 3*4 - 12*2 + 5 = -7$$

vertex coordinates : $(2, -7)$.

Answer: $(2, -7)$

4. What is the expected value $E(2X)$ of a random variable with distribution:

| | | | | |
|------|-----|-----|-----|--|
| X | 1 | 2 | 3 | |
| P(X) | 0.2 | 0.5 | 0.3 | |

- ☐ 1.8
- ☐ 4.2
- ☐ 2.1
- ☐ 2.5

We know that,

$$E[2X] = 2E[X]$$

$$E[X] = 1 \times 0.2 + 2 \times 0.5 + 3 \times 0.3 = 2.1$$

$$E[2X] = 2E[X] = 2 \times 2.1 = 4.2$$

Answer: 4.2

5. A geometric sequence has first term $a = 4$ and ratio $r = 2$. Find the 5th term.

- ☐ **64**
- ☐ **32**
- ☐ **16**
- ☐ **128**

$$u_n = u_1 r^{n-1}$$

$$u_n = 4 \times 2^{n-1}$$

$$u_5 = 4 \times 2^{5-1} = 4 \times 16 = 64$$

Answer: 64

6. The roots of the equation $x^2 - 4x + 13 = 0$ are:

- ☐ $-2 \pm 3i$
- ☐ $2 \pm 3i$
- ☐ $2 \pm \sqrt{13}i$
- ☐ $4 \pm 3i$

We use the resolving formula to solve the equation $x^2 - 4x + 13 = 0$

$$x = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 13}}{2 \times 1} \Leftrightarrow x = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 13}}{2} \Leftrightarrow x = \frac{4 \pm \sqrt{-36}}{2} \Leftrightarrow$$
$$x = \frac{4 \pm \sqrt{36i^2}}{2} \Leftrightarrow x = \frac{4 \pm 6i}{2}$$

$$\Leftrightarrow x = 2 \pm 3i$$

Answer: $x = 2 \pm 3i$

7. Find the derivative of $f(x) = x^3 + 2x \ln(x)$ at $x = 1$:

- ☐ **3**
- ☐ **2**
- ☐ **5**
- ☐ **4**

$$f(x) = x^3 + 2x \ln(x)$$

$$f'(x) = 3x^2 + 2 \ln(x) + 2$$

$$f'(1) = 3 * 1 + 2 \ln(1) + 2 = 5$$

Answer: 5

8. The horizontal asymptote of the function $f(x) = (4x^2 + 1)/(2x^2 - 5)$ is:

- ☐ $y = 0$
- ☐ $y = 1$
- ☐ $y = 2$
- ☐ $y = 4$

We divide $(4x^2 + 1)/(2x^2 - 5)$

$$(4x^2 + 1)/(2x^2 - 5) = 2 + 11/(2x^2 - 5)$$

$y = 2$ is a horizontal asymptote

Answer: $y = 2$

10. Convert the complex number $Z = -3 + 3i$ into polar form (modulus and angle):

- ☐ **$3\sqrt{2}, 3\pi/4$**
- ☐ **$3\sqrt{2}, 5\pi/4$**
- ☐ **$6, \pi/4$**
- ☐ **$3, 3\pi/4$**

$$r = \sqrt{(-3)^2 + (3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$$

$$\text{Tg}(\text{teta}) = -3/3 = -1 \quad \text{teta} = 3\pi/4$$

Answer: $d = 3\sqrt{2}, 3\pi/4$