CATÓLICA FACULDADE DE MEDICINA
LISBOA

UNIVERSIDADE CATÓLICA PORTUGUESA

Mathematics

preparing students for exams

Ana Barqueira| june 5, 2023

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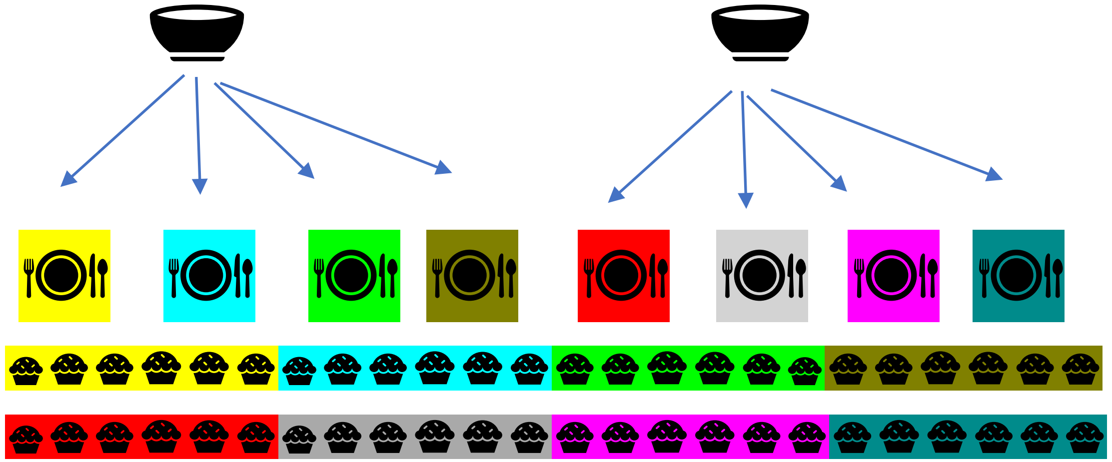
1. **For the menu of a graduation dinner, we must choose between two soups, four main dishes and six desserts. How many are the choices of a menu with one soup, one main dish and one dessert?**

To compute all the possible menus with 1 soup, one main dissh and one dessert, we only have to do

2x4x6 = 48



Answer: 48 (b)

2. **Consider X the random variable "number of times you go shopping per month". The following table corresponds to the probability distribution of variable X:**

X = x i	0	1	2	3	4
P(X = xi)	2k	0,26	0,2	0,15	k/2

The value of P(X=0) is:

We should start by compute the value of K.
To do this, we use the property that says: the sum of values of a probabiliy function is equal to 1

$2k + 0,26 + 0,2 + 0,15 + k/2 =1$

$5k/2 + 0,61 = 1$

$5k/2 = 1-0,61$

$5k/2 = 0,39$

$5K = 0,39x2$

$K = (0,39x2)/5$

$K = 0,156$

Then, $P(X=0) = 2k = 2x0,156 = 0,312$

Answer: 0,312 (d)

3. It is known that: $P(AUB) = 0{,}68$, $P(B) = 0{,}42$ e $P(A) = 0{,}5$. Calculate $P(A|B)$.

We know that,
$P(AUB) = P(A) + P(B) - P(A\cap B)$
$P(A|B) = P(A\cap B)/P(B)$

1. Compute $P(A\cap B)$
   $0{,}68 = 0{,}5 + 0{,}42 - P(A\cap B)$
   $P(A\cap B) = 0{,}5 + 0{,}42 - 0{,}68 = 0{,}24$

2. Compute $P(A|B)$
   $P(A|B) = 0{,}24/0{,}42 = 0{,}571$

Answer: 0,571

4. **In a university it is known that:**

- the number of students speaking French is equal to the number of students speaking Spanish;

- the number of students speaking one of the languages is three times the number of students speaking French and Spanish.

If you randomly choose a student from this school, what is the probability that this student will speak Spanish, knowing that he or she speaks French?

Let **F** = "student speaking French" and **S** = "student speaking Spanish".

P(F) = P(S)

$P(S \setminus F) + P(F \setminus S) = 3P(F \cap S) \Leftrightarrow P(S) - P(F \cap S) + P(F) - P(F \cap S) = 3(F \cap S) \Leftrightarrow$

$\Leftrightarrow P(S) + P(F) - 2P(F \cap S) = 3(F \cap S) \Leftrightarrow P(S) + P(F) = 5P(F \cap S) \Leftrightarrow 2P(F) = 5P(F \cap S) \Leftrightarrow P(F) = 5/2P(F \cap S).$

We want to calculate P(S|F) and by definition,

$$P(S|F) = \frac{P(F \cap S)}{P(F)} = \frac{P(F \cap S)}{5/2P(F \cap S)} = 2/5.$$

Answer: 2/5

5. If α is an angle of an equilateral triangle, the value of the expression ***$tg(\alpha)cos^2(\alpha)+tg(\alpha)sen^2(\alpha)$*****, is:**

We know that all the angles of an equilateral triangle have amplitude equal to 60°, so $\alpha = 60°$.

We can write the expression **$tg(\alpha)cos^2(\alpha)+tg(\alpha)sen^2(\alpha)$** in an equivalent form that is,

$tg(\alpha)cos^2(\alpha)+tg(\alpha)sen^2(\alpha) = tg(\alpha)(cos^2(\alpha)+ sen^2(\alpha)) = tg(\alpha)x1 = tg(\alpha)$

So, as $\alpha = 60°$, we have that $tg(60°) = \sqrt{3}$

So, the value of $tg(\alpha)cos^2(\alpha)+tg(\alpha)sen^2(\alpha) = \sqrt{3}$

Answer: $\sqrt{3}$ (b)

6. The equation $x^2+6x+y^2-2y=15$ represents a circle. What are the coordinates of its centre?

We know that the equation of a circle is $(x-c_1)^2+(x-c_2)^2=r^2$

We can rewrite the equation to form the expression of the square of the binomial for *x* and for *y,* like

$(x^2+6x+9)+(y^2-2y+1)=15+9+1 \Leftrightarrow$

$(x+3)^2+(y-1)^2=25.$

The expression above is the equation of a circle of centre (-3,1) and radius equal to 5.

Answer: C = (-3,1)

7. Let t be a line whose slope is m = 2/5. Knowing that the line s is perpendicular to the line t, its slope is:

We know that if we have a line with slope m, then a line perpendicular to it have slope -1/m.

The slope of line t is 2/5.

If the line s is perpendicular to line t, it slope, applying the property above is -5/2

Answer: -5/2 (d)

8. Simplify the expression $w = \frac{3(2+i)}{(4-i)(4+i)}$, applying mathematical operations to complex numbers.

$$w = \frac{3(2+i)}{(4-i)(4+i)} = \frac{3(2+i)}{16-i^2} = \frac{3(2+i)}{16+1} = \frac{6+3i}{17}$$

Answer: $w = \frac{6+3i}{17}$

9. One of the solutions of the equation x^2 - 4x + 8 = 0 is:

We use the resolving formula to solve the equation x^2 - 4x + 8 = 0

$$x = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 8}}{2 \times 1} \Leftrightarrow x = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 8}}{2} \Leftrightarrow x = \frac{4 \pm \sqrt{-16}}{2} \Leftrightarrow x = \frac{4 \pm \sqrt{16 i^2}}{2} \Leftrightarrow x = \frac{4 \pm 4i}{2}$$

$$\Leftrightarrow x = 2 \pm 2i$$

Answer: $\mathbf{2 + 2i \ (c)}$

10. Present the complex $z = 7e^{i(\frac{\pi}{4})}$ in algebraic form.

$$z = 7e^{i(\frac{\pi}{4})} = 7\cos(\frac{\pi}{4}) + 7\text{sen}(\frac{\pi}{4})i = \frac{7\sqrt{2}}{2} + \frac{7\sqrt{2}}{2}i$$

Answer: $z = \frac{7\sqrt{2}}{2} + \frac{7\sqrt{2}}{2}i$

11. The value of the expression $2sen\left(\frac{\pi}{3}\right)+3tg\left(\frac{\pi}{6}\right)+2cos\left(\frac{\pi}{4}\right)+sen(\pi)$ is:

$sen\left(\frac{\pi}{3}\right)$ = $\sqrt{3}/2$
$tg\left(\frac{\pi}{6}\right)$ = $\sqrt{3}/3$
$cos\left(\frac{\pi}{4}\right)$ = $\sqrt{2}/2$
$sen(\pi)$ = 0

So,
$2sen\left(\frac{\pi}{3}\right)+3tg\left(\frac{\pi}{6}\right)+2cos\left(\frac{\pi}{4}\right)+sen(\pi)$ = 2x $\sqrt{3}/2$ + 3x $\sqrt{3}/3$ + 2x $\sqrt{2}/2$ + 0 =2 $\sqrt{3}$ + $\sqrt{2}$

Answer: 2 $\sqrt{3}$ + $\sqrt{2}$ **(c)**

12. State the solution of the inequality $\frac{x^2-4}{1+3x} > 0$

$x^2 - 4 = 0 \Leftrightarrow x^2 = 4 \Leftrightarrow x = \pm\sqrt{4} \Leftrightarrow x = \pm 2$

$1 + 3x = 0 \Leftrightarrow x = -1/3$

	$-\infty$		-2		$-1/3$		2		$+\infty$
x^2-4		+	0	-		-	0	+	
$1+3x$		-		-	0	+		+	
$f(x)$		-	0	+	n.d	-	0	+	

$x \in \left]-2, -1/3\right[\cup \left]2, +\infty\right[$

Answer: $x \in \left]-2, -1/3\right[\cup \left]2, +\infty\right[$

13. Consider the function $f(x) = \frac{3x-4}{x-5}$ in $\mathbb{R}$. Indicate all asymptotes of the function *f(x)*.

If we devide the numerator by the denominatour of f(x) we have

(3x -4)/ (x-5) = 3 + $\frac{11}{x-5}$

So, the function has a vertical asymptote at *x = 5* and a horizontal asymptote at *y = 3*

Answer: $x = 5, y = 3$

14. The point corresponding to the maximiser of the function $f(x) = -24x + 3\ln(x)$ is:

To compute the maximising point of function f we have to calculate the first derivative and compute its zeros.

f'(x) = -24 +3/x
f'(x) = -24 +3/x = 0 ⟺ -24x +3 = 0 ∧ x ≠ 0 ⟺ x = 3/24 ∧ x ≠ 0 ⟺ x = 1/8 ∧ x ≠ 0

To confirm that 1/8 is a maximiser of function f we have to compute the second derivative of function f and verify that its value is always negative.

f''(x) = $\frac{-3}{x^2} < 0$

Answer: $\boldsymbol{x = \frac{1}{8}\ \ (a)}$

15. Calculate the value of the second derivative of the function $f(x)$ = 4x^3 - 3ln(x) at the point *x=3*.

f'(x) = $12x^2 - \frac{3}{x}$

f''(x) = $24x + \frac{3}{x^2}$

f''(3) = 24x3 + 3/9 = 217/3

Answer: $\boldsymbol{x = \frac{217}{3}}$

16. Consider the following function of domain $\mathbb{R}^+$: $g(x) = -5.\,ln(x^2)$. The 1st derivative of the function $h(x)=[g(x)^2]$:

h'(x) = 2.g(x).g'(x)

h'(x) = $2(-5.\,ln(x^2)).(-5.\,ln(x^2))'$

$h'(x) = -10\ln(x^2).(-5\frac{2x}{x^2})$

$h'(x) = 50\ln(x^2).\left(\frac{2}{x}\right) = \frac{100}{x}\ln(x^2)$

If $x \in]0,1[\,, \frac{100}{x}\ln(x^2) < 0$

If $x \in]1,+\infty[\,, \frac{100}{x}\ln(x^2) > 0$

Answer: It is positive or negative **(a)**

17. Consider the function: $p(x)$ = $10x^2 - e^{3x}$. The abscissa of the inflection point of this function is:

To compute the inflection point of a function we have to calculate the second derivative and equal the second derivative to zero.

p'(x) = $20x - 3e^{3x}$

p''(x) = $20 - 9e^{3x}$

p''(x) = $20 - 9e^{3x} = 0 \Leftrightarrow 9e^{3x} = 20 \Leftrightarrow e^{3x} = \frac{20}{9} \Leftrightarrow 3\text{x} = \ln(\frac{20}{9}) \Leftrightarrow \text{x} = \ln(\frac{20}{9})/3$

	$-\infty$		$\ln(\frac{20}{9})/3$		$+\infty$
$20 - 9e^{3x}$		+	0	-	
p(x)		∪	I.P.	∩	

Answer: $\text{x} = \ln(\frac{20}{9})/3$ **(b)**

18. Calculate the value of the following limits.

a) $\lim_{x\to0} \frac{2x}{e^{3x}-1}$ b) $\lim_{x\to0} \frac{sen4x}{5x}$

$$\lim_{x\longrightarrow0} \frac{2x}{e^{3x}-1} = \lim_{x\longrightarrow0} \left(\frac{e^{3x}-1}{2x}\right)^{-1} = \frac{2}{3}\lim_{x\longrightarrow0} \left(\frac{e^{3x}-1}{3x}\right)^{-1} = \frac{2}{3} \times 1^{-1} = \frac{2}{3}$$

$$\lim_{x\longrightarrow0} \frac{sen(4x)}{5x} = \frac{1}{5}\lim_{x\longrightarrow0} \left(\frac{sen(4x)}{x}\right) = \frac{4}{5}\lim_{x\longrightarrow0} \left(\frac{sen(4x)}{4x}\right) = \frac{4}{5} \times 1 = \frac{4}{5}$$

Answer: $\lim_{x\longrightarrow0} \frac{2x}{e^{3x}-1} = 2/3$ $\lim_{x\longrightarrow0} \frac{sen(4x)}{5x} = 4/5$

19. Let *f(x)* be a function of domain $\mathbb{R}^+$. It is known that $\lim_{x\to+\infty} \frac{x}{f(x)} = 3$ and $\lim_{x\to+\infty} [f(x) - \frac{1}{3}x] = 4$.
The equation that defines an asymptote of the graph of the function *f(x)* is:

If $\lim_{x\to+\infty} \frac{x}{f(x)} = 3$, the slope (m) of an oblique asymptote is 1/3 $\left(\lim_{x\to+\infty} \frac{f(x)}{x} = \frac{1}{3} \right)$

If $\lim_{x\to+\infty} [f(x) - \frac{1}{3}x] = 4$, the value of term independent (b) of an oblique asymptote is 4.

So, the oblique asymptote of function f is $y = \mathrm{mx} + \mathrm{b} = \frac{1}{3}x + 4$

Answer: $y = \frac{1}{3}x + 4$ **(c)**

20. Calculate the solution set of the equation $\frac{1}{27^{(x-5)}} - 3^{(x+4)} = 0$.

$$\frac{1}{27^{(x-5)}} - 3^{(x+4)} = 0 \Leftrightarrow \frac{1}{27^{(x-5)}} = 3^{(x+4)} \Leftrightarrow \frac{1}{3^{3(x-5)}} = 3^{(x+4)} \Leftrightarrow 3^{-3(x-5)} = 3^{(x+4)} \Leftrightarrow -3(x-5) = (x+4)$$

$$-3x + 15 = x + 4 \Leftrightarrow -4x = -11 \Leftrightarrow x = 11/4$$

Answer: x = 11/4